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Unit Radius and the Resolution of Inverse Square Law Forces

J. C. BELCHER

63 Avondale Road, Shipley, West Yorkshire, BD18 4QU

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Abstract

An equation for the solution of inverse square law forces that arise in rectilinear systems from gravitational, electrical, and magnetic fields, produced by complex multibody or multielement sources, is derived from a concept known as *unit radius*, which is associated with orbital systems, and the use of this equation in computer evaluation of simple models such as disks and spheres shows results that are in good agreement with those predicted by classical methods using a somewhat indirect approach. Additionally, these results show that, within the profile of the model so analyzed, the effective distance between the reference point and the resultant source is equal in all cases to the radius of the model, and that—again within the profile of the model—the magnitude of the resultant source varies directly with the distance of the reference point from the center of the model. The *proximity effect* of sources of appreciable dimensions is examined in some detail and there is good evidence to show that as a result of this effect the resultant force when measured at the perimeter of a two-dimensional disk-shaped source is some 23% lower than the value anticipated by classical theory.

1. *Introduction*

Present day understanding of inverse square law forces, whether they be gravitational, electrical, or magnetic, is governed to a very large extent by Newton's concept of gravitational force as it applies to the Earth-Moon system.

This system, unique in the solar system has three important limitations:

1.1.1. It is essentially a point-source system. That is to say the radii of the Earth and Moon are seen to be very small when compared with the distance separating the two bodies. Because of this, such factors as *proximity effect* and *profile effect* are not of sufficient magnitude to alter to any significant extent the force relationships that prevail.

1.1.2. It is a single-element system in that it has one primary body and a single secondary body that orbits the primary. Under these special circumstances the general situation is considerably simplified and bears little resemblance to the situation that would exist in a multielement system.

1.1.3. It is an orbital system, and therefore basically a closed independent system of a particular order that can be influenced or acted upon only by the primary bodies of higher-order systems. Thus certain relationships that appear to hold good in a rectilinear system do not apply.

These limitations, underlining as they do the very basic simplicity of the Earth–Moon system, must yet be recognized as the key factors that lead to Newton's initial understanding of gravitational force. It follows in turn, however, that Newton's concept itself must necessarily be simple in form and content and as such limited in its application to all but the simplest of problems, for example the following:

1.2.1. Those involving point sources only and in which the *proximity* of the source and its shape or profile factor is relatively unimportant. In the long term, such a restriction does not make for a complete understanding of, for example, the effect of so-called "points" on the electrical charge intensity of a source of electrical force.

1.2.2. Those involving single elements. Because there is no logical chain connecting single-element and multielement systems, it follows that a solution to the so-called three-body or multibody problem is unlikely to stem directly from Newton's concept.

1.2.3. Those involving orbital systems only. This constraint is of less immediate importance that 1.2.1 or 1.2.2 and becomes significant only when the tidal forces applying to the primary of a system are being considered. The distinction between orbital systems and rotational systems –in the sense that the Earth is a system that rotates–is not at present a clearly defined one.

It follows from this that any progress made towards gaining a working knowledge of complex situations involving inverse square law forces will stem only from a prior familiarization of the mechanics of multielement orbital systems, that is from a basic understanding of the solar system itself. Unfortunately, experience has shown that such understanding cannot emerge either logically or intuitively from the traditional Earth–Moon concept. This is because the latter concept is but a particular application of a more general principle.

What a continuing study of orbital space systems does show, however, is that in any orbital system the effectiveness of certain necessarily interdependent relationships, both individually and collectively, hinges on one common concept of paramount importance, a specific orbital radius of unit length. It is largely because of this factor that the author has come to identify the term *unit radius* with the whole of this particular concept in mind, more particularly so since the chosen term readily lends itself to describing a wide spectrum of situations typical of orbital systems.

In the widest sense, implicit in the concept of *unit radius* is the basic recognition that this particular radius itself defines the fundamental *unit of length* for the particular system concerned. It can be shown that used in this application, therefore, *unit radius* forms a powerful basis for a system of comparative cosmology, and moreover brings a whole new significance to dimensional analysis.

In a narrower sense, *unit radius* is an integral factor in an equation describing the mass–radius relationships of orbiting bodies and has therefore a close association with the understanding of the gravitational forces involved in a multielement system.

In this paper the author describes the development and use of a derived equation that can be applied to the solution of problems involving inverse square law forces in rectilinear systems, both when these forces arise from multielement sources and when they exist in multibody situations. That is to say, the equation overcomes the limitations of Newton's concept as discussed in 1.2.1 and 1.2.2.

Analysis of simple models using the equation in a computer evaluation shows that, in the case of spherical or circular arrays of elements, results are in good agreement with those anticipated by traditional methods using a somewhat indirect approach. Additionally, it is shown that the following conditions obtain:

1.3.1. The resultant distance between any *internal* reference point and the resultant source of the force exerted by the sphere or disk is equal in all cases to the radius of the sphere or disk.

1.3.2. The resultant magnitude of the source *within* the surface of the sphere or *within* the perimeter of the disk varies directly with distance from the centre of the sphere or disk, being zero at the centre and maximum at the surface or perimeter.

One significant factor that is brought to light by such analysis, and which is subsequently examined in some detail, is that of *proximity effect.* This is seen to be most serious at the surface of a sphere or at the perimeter of a disk, and is identified by a considerable reduction in the effective magnitude of the source—and thus in turn in the value of the resultant force—as experienced at that point.

Profile effect (1.2.1) and *tidal effect* (1.2.3) both demand extensive analysis and are not examined in the present paper. *Profile effect* as defined by the author is proximity effect in a modified form, and is determined by the extent by which the shape of a source departs from the truly spherical or circular. An understanding of profile effect can be important in that it leads to the determination of *ideal* source profiles. *Tidal effect* includes the relatively minor *undulatory tidal force* experienced in single-element systems, and *cataclysmic tidal force*—many orders of magnitude greater—which is superimposed on undulatory tidal force and is experienced only in multielement systems. Cataclysmic tidal force on the Sun, for example, would appear to be a major factor in the production of *irruptive forces* resulting in sunspots, and *eruptive forces* resulting in other forms of solar activity.

2. *Development and Use of the Basic Equation*

It can be shown that any complex multielement orbital system can be replaced by a simple theoretically equivalent single-element system in which the following conditions hold:

2.1.1. The mass of the secondary element in the equivalent system is equal to the sum of the masses of the individual elements in the multielement system.

2.1.2. The orbital radius of the secondary element in the equivalent system is equal to the geometric mean of the orbital radii of the individual elements in the multielement system.

2.1.3. The orbital radius of the secondary element in the equivalent system is of unit length.

It follows from this that the resultant inverse square law force, $\mathbf{F_n}$, between a primary and its n secondaries is

$$\mathbf{F_n} = \mathbf{k}M_0 \Bigg/ \left\{ \prod_{x=1}^{n} r_x \right\}^{2/n} \sum_{x=1}^{n} M_x \cdots \tag{2.1}$$

where $\mathbf{k}$ is a dimensional constant, M_0 is the mass of the primary body, M_x is the mass of the xth secondary element, r_x is the orbital radius of the xth secondary element, and r_u is the *unit radius*,

$$r_u = \left\{ \prod_{x=1}^{n} r_x \right\}^{1/n}$$

The use of *unit radius*, a unique standard of length, shows that this is a unique dynamic self-contained self-maintained system or cosmos, in which each component is necessarily subject to certain space-time, mass-radius relationships that are peculiar to the system itself.

By contrast, in examining force relationship that exist in a static rectilinear system due allowance must be made for any space-mass relationships that apply, and the ensuing modified *basic* equation, divorced of its original association with gravitation and unit radius, follows:

$$\mathbf{F_n} = \mathbf{k}A_0 \Bigg/ \left\{ \prod_{x=1}^{n} r_x^{(A_x \cos\theta_x / \Sigma A_x \cos\theta_x)} \right\}^2 \sum_{x=1}^{n} A_x \cos\theta_x \cdots \tag{2.2}$$

where A_0 is the magnitude of the reference point expressed in terms of unit charge, unit pole, unit mass, etc., A_x is the magnitude of the xth individual point source, r_x is the distance between A_x and the reference point A_0, and θ_x is the angle subtended by r_x to the reference vector at A_0. It will be seen that when $n = 1$, then:

$$\mathbf{F_1} = (\mathbf{k}A_0/r_1{}^2)A_1 \cos\theta_1$$

and this is the only link with traditional theory.

Because of these modifications, when applying equation (2.2) to any problem it becomes necessary to abide by three very important working rules:

2.2.1. Rule 1: The basic equation is applicable only to situations prevailing in a closed system or cosmos.

2.2.2. Rule 2: The unit of length employed must be such that r_x is not equal to unity.

2.2.3. Rule 3: The unit of length employed must be such that the resultant distance, r_0, is not equal to unity.

Experience gained in applying equation (2.2) to even simple problems demonstrates to the user a subtle break from certain poorly conceived traditional thinking. It is at first confusing, for example, to discover that in an in-line three-body problem the force exerted by a source of magnitude 9 units at a distance of 3 unit lengths is somewhat greater than a second source of magnitude 4 units at a distance of 2 unit lengths, until it is recognized that the magnitude of a source is of greater importance than its position in space:

$$\mathbf{kF}_2 = \frac{(9-4)}{(3^{9/5} \times 2^{-4/5})^2}$$

$$= 0.29036$$

It follows, therefore, that continued use of the basic equation underlines in no uncertain manner the degree of balance and harmony that lay behind natural laws.

As indicated above, the equation is simple to apply and is amenable to a variety of operational techniques. Multisource or multibody problems can be resolved by part, i.e., in a number of simple stages, or by whole, i.e., in one operation. In the odd unforeseen case, exponents may be readily resolved by the use of the $e^{0.1x}$ or the e^x and x scales of the slide-rule, whilst complex problems can be made to respond to solution by relatively simple computer programs. It is this latter aspect that will be examined next.

3. *Computer Evaluation of Simple Models*

Fairly complex problems involving inverse square law forces can readily be solved by using the basic equation in relatively simple computer programs. By this means, simple models such as multielement disks and multielement spheres have been analyzed in order to determine the forces and other conditions that obtain. The procedure that is followed in the case of multielement disks can best be understood by reference to Figure 1 and Appendices 1 and 2.

Appendix 1 shows a listing of a BASIC program, PROG1, which is used to generate the data to describe a quadrant of a multielement disk. When the program is run, an early command is given that determines the number of elements or squares, N, per radius, r. The magnitude of each source, whether this be mass, electric charge, or magnetic pole, is determined by the parameter w, this being the measure of the area of each square or part of a square. A subprogram looks at the conditions applying to parts of squares and classes them into one of four groups, thence calculating their areas by using the midordinate rule or else by approximating them to triangular areas. The location of each element is given by parameters x and y, and once again in the case of parts of squares an approximation is made. The data file, WFILE, when complete thus contains the data in the form w, x, y. Finally, the program prints out the number of elements in the data file, the calculated area of the resulting circular

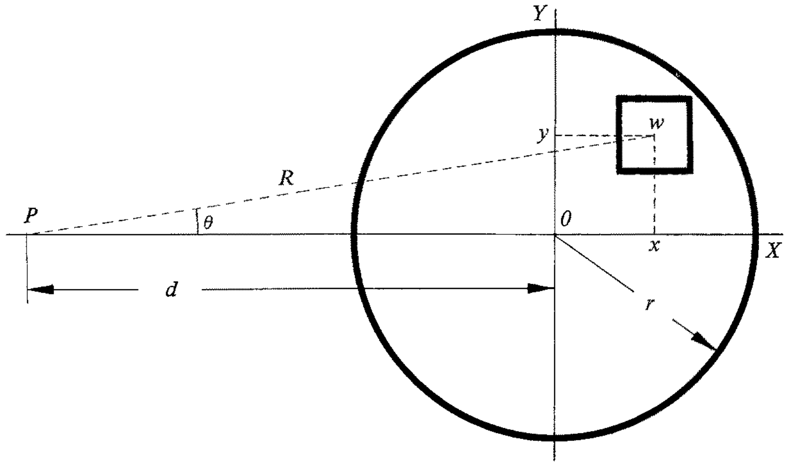

Figure 1—Computer evaluation of the resultant force and other conditions applying to a finite array of sources arranged in the form of a multielement disk.

disk, which approximates to 3.14159, and lastly the error factor, which is a measure of the accuracy of the data generated. Typical error factors are given in Table 3 and it will be seen that for a disk composed of 224 elements this factor is 1.000430, i.e., an overall error of +0.043%.

The data file for a quadrant in which $N = 2$, resulting in a circular disk of 16 elements, is given in Table 1, the quadrant being illustrated in Figure 2. The circle error factor in this instance is 0.999791, i.e., an overall error of −0.0209%, showing that in an extreme case prone to large errors the accuracy of the generated data is at a consistently high level.

When the second program, PROG2, is run, an early command determines the distance, d, in terms of the radius, r, between the centre of the disk, O, and the reference point P, this reference point in effect identifying the location of point source, A_0, in equation (2.2). The data are then read, and the following calculations made for each set of data:

$$R = [(x + d)^2 + y^2]^{1/2}$$

$$\cos\theta = (x + d)/R$$

$$A \cos\theta = w(x + d)/R$$

As the data are being read, all values of $A \cos\theta$ are added together to give a printout, $\Sigma A \cos\theta$, the summation of all values of $A \cos\theta$. The data are then read for a second time, the relevant calculation being

$$L = R^{(A \cos\theta/\Sigma A \cos\theta)}$$

TABLE 1.

Element	w	x	y
1	1.000000	0.250000	0.250000
2	0.936492	0.734123	0.250000
3	0.936492	0.250000	0.734123
4	0.267949	0.566987	0.566987

As the data are read, all values of L are multiplied together to give the ultimate value of R_0; the final printout giving the value of R_0, the value of the "force" = $\Sigma A \cos\theta / R_0^2$, and the factor, R_0/d.

In use, the reference point P can be set at any position external or internal to the disk; from infinity, through r, to zero at the centre. By running the program for a number of different positions of P, it is possible to examine the conditions that apply in any, including the extreme, case.

Typical results, obtained from a disk of 224 elements, are given in Table 2. These are shown in terms of the maximum values. Corresponding curves of R_0, $\Sigma A_x \cos\theta_x$, and the resultant force, all related to distance from the centre of the source, are given in Figure 3.

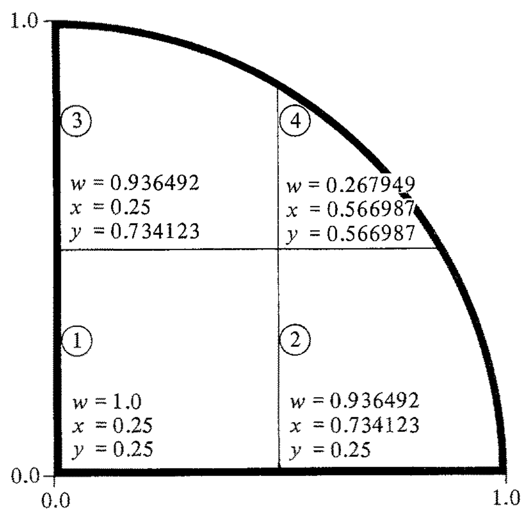

Figure 2 – A quadrant of a circular disk in which the number of elements per radius, N, is equal to 2. Element data are computer generated and the figures may suggest an accuracy that is not factual.

TABLE 2.

Distance (d) $r = 1$	R_0 $r = 1$	R_0/d	$\Sigma A_x \cos\theta_x$ $\Sigma A_x = 1$	Force ($\Sigma A_x \cos\theta_x / R_0^2$) max = 1
1000	1000	1.00000	*1.000000*	0.000001
100	100	1.00000	0.999990	0.000129
10	10	1.00000	0.998757	0.012858
6	6.00010	1.00002	0.996535	0.035636
4	4.00033	1.00008	0.992165	0.079820
2.25	2.25203	1.00090	0.974795	0.247447
2	2.00299	1.00149	0.967864	0.310581
1.75	1.75469	1.00268	0.957554	0.400385
1.5	1.50812	1.00541	0.941158	0.532733
1.35	1.38628	1.00820	0.928978	0.622334
1.25	1.26656	1.01325	0.912313	0.732172
1.125	1.15152	1.02357	0.888331	0.862483
1.075	1.10835	1.03102	0.875495	0.917532
1.05	1.08796	1.03615	0.868057	0.944151
1.025	1.06892	1.04285	0.859740	0.968715
1.000	1.05214	1.05214	0.850269	0.988838
0.975	1.03942	1.05214	0.839203	*1.000000*
0.95	1.03296	1.08732	0.826083	0.996736
0.925	1.03118	1.11478	0.811313	0.982297
0.875	1.02080	1.16663	0.781211	0.965176
0.75	1.00913	1.34550	0.693390	0.876602
0.625	1.00383	1.60612	0.593280	0.757978
0.5	1.00145	2.00289	0.484198	0.621563
0.375	1.00050	2.66800	0.368525	0.473971
0.25	1.00022	4.00086	0.248176	0.319368
0.125	1.00018	8.00146	0.124824	0.160641
* 0.1	0.97640	9.76403	0.100561	0.135796
* 0.05	1.03318	20.6636	0.049594	0.059814
* 0.01	1.13011	113.011	0.009695	0.009773

* Computer rounding error attains a large positive value when $(r/N > d > (r/2N)$, and a large negative value when $(r/2N) > d > 0$.

4. *Proximity Effect of Finite Sources*

As shown in Table 2, at distances far removed from the centre of a source, the effective magnitude of the resultant source, $\Sigma A_x \cos\theta_x$, is sensibly equal

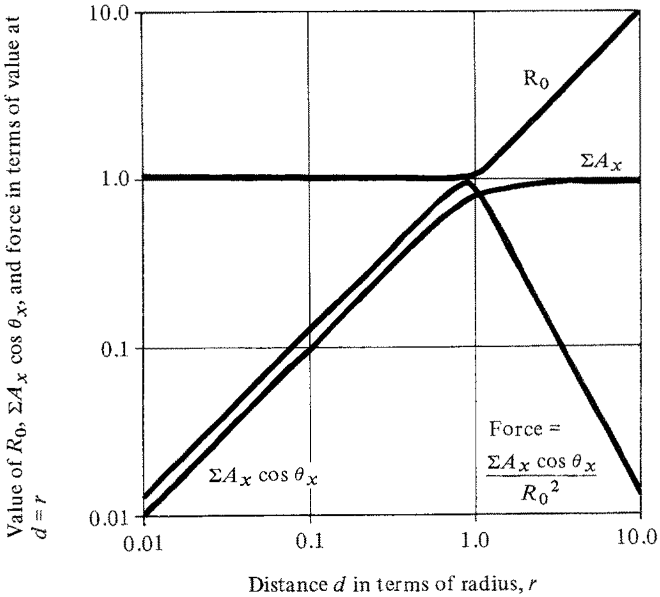

Figure 3—Curves of R_0, $\Sigma A_x \cos\theta_x$, and the resultant force, related to distance from the center of a disk or sphere, $\Sigma A_x \cos\theta_x$ tends to the value 0.8502 at a distance equal to r, the radius of the source.

to the maximum value, ΣA_x. At the centre of the resultant source, assuming this to be symmetrical about the centre, $\Sigma A_x \cos\theta_x = 0$.

Between these extremes the general pattern presented by the data suggests that at distances where $d \geqslant r$, $\Sigma A_x \cos\theta_x = \Sigma A_x$, i.e., the maximum value, and at distances where $r > d > 0$, $\Sigma A_x \cos\theta_x$ varies directly with d, being zero at the centre and maximum at the surface.

In practice, where the source is of appreciable dimensions, the actual values encountered are somewhat less than the ideal values anticipated. This inconsistency would appear to be caused by the very proximity of the source itself, since what might be termed *proximity effect* only becomes significant in the near vicinity of the surface of the source.

Analysis of three-dimensional solids, as distinct from two-dimensional disks, using a modified computer program, has shown that there is no significant difference in the results obtained from a sphere of N-elements/radius and a disk of N-elements/radius, the increased "bluntness" of the source merely increasing proximity effect. Thus shape, in this sense, has no direct bearing on the final result.

As shown in Table 3 and Figure 4, the number of elements that go to make up the source can affect the result to some extent, especially when only a small number of elements is considered. When the number of elements is increased, however, the ratio $\Sigma A_x \cos\theta_x / \Sigma A_x$ at the perimeter of the disk tends towards

TABLE 3.

Divisions per radius	Number of elements	Circle error factor	ΣA_x $d = 100r$	$\Sigma A_x \cos\theta_x$ $d = r$	$\frac{\Sigma A_x \cos\theta_x}{\Sigma A_x}$
2	16	0.999791	12.56372	10.8596	0.864361
3	36	0.997721	28.2099	24.1694	0.856770
4	60	1.001910	50.36160	42.9731	0.853290
6	132	0.999374	113.02632	96.258	0.851642
8	224	1.000430	201.146	171.630	0.850269
10	352	0.999745	314.079	267.046	0.850251

the value 0.8502, and this would seem to be representative of the value to be expected from a disk composed of an infinite number of elements.

The basis of proximity effect would appear to lie in the fact that as the reference point, *P*, in Figure 1 approaches the surface of the sphere or the perimeter of the disk, the angle θ, subtended to the reference vector *PO* by those elements nearly tangenital to the surface or to the perimeter will approach 90°, and therefore in each case $A_x \cos\theta_x$ will approach zero, and it follows that $\Sigma A_x \cos\theta_x$ must be less than the maximum value ΣA_x.

As shown in Table 4 and Figure 5, the error in $\Sigma A_x \cos\theta_x$, introduced by proximity effect, in the case of a two-dimensional disk has a maximum value

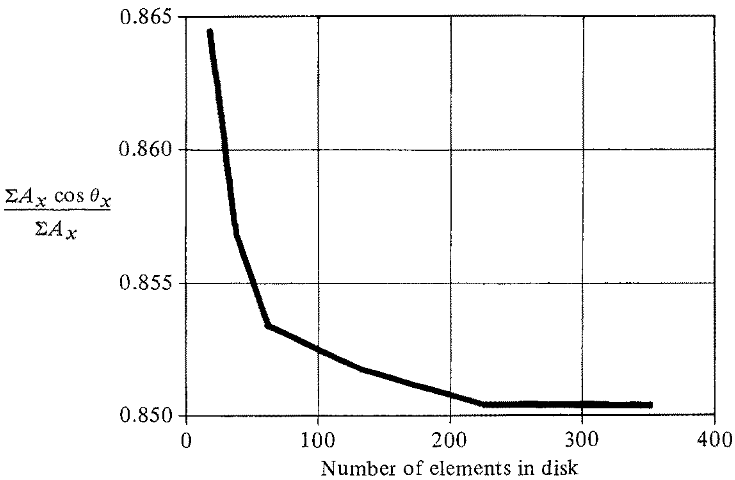

Figure 4 – The relationship between the value of $\Sigma A_x \cos\theta_x$ at a distance equal to the radius of a disk, and the number of elements that go to make up such a disk. The value tends towards 0.8502 for a large number of elements.

of some 15% at the perimeter. Moreover, any negative error in $\Sigma A_x \cos \theta_x$ is reflected as a positive error in R_0, and this is shown to have a maximum value of some 5% at the perimeter of the disk.

TABLE 4.

Distance (d) in terms of r	Error in $\Sigma A_x \cos \theta_x$ due to proximity effect (negative)	Error in R_0 due to error in $\Sigma A_x \cos \theta_x$ (positive)
1000	0.000000	0.00000
100	0.000010	0.00000
10	0.001243	0.00000
6	0.003465	0.00002
4	0.007835	0.00008
2.25	0.025205	0.00090
2	0.032136	0.00149
1.75	0.042446	0.00268
1.5	0.058842	0.00541
1.35	0.071022	0.00820
1.25	0.087687	0.01325
1.125	0.111669	0.02357
1.075	0.124505	0.03102
1.05	0.131943	0.03615
1.025	0.140260	0.04285
1.000	*0.149731*	*0.05214*
0.0975	0.135797	0.03942
0.95	0.123917	0.03296
0.925	0.113687	0.03118
0.875	0.093789	0.02080
0.75	0.056610	0.00913
0.625	0.031720	0.00383
0.5	0.015802	0.00145
0.375	0.006475	0.00050
0.25	0.001824	0.00022
0.125	0.000176	0.00018
* 0.1	+0.000561	–0.023597
* 0.05	0.000406	0.03318
* 0.01	0.000305	0.13011

* Computer rounding error attains a large positive value when $(r/N) > d > (r/2N)$, and a large negative value when $(r/2N) > d > 0$.

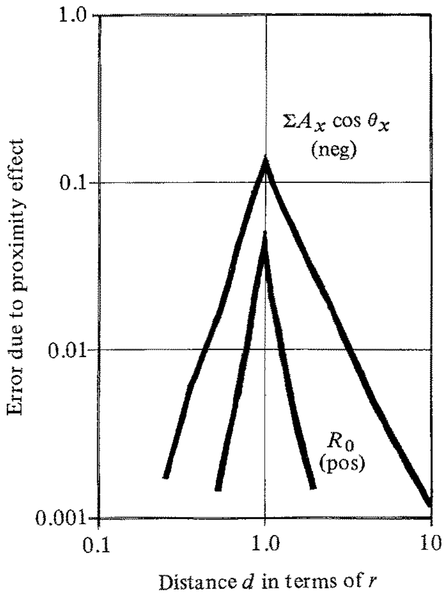

Figure 5—The errors introduced by proximity effects are negative in the case of $\Sigma A_x \cos \theta_x$, and positive in the case of R_0.

It follows that if $\Sigma A_x \cos \theta_x = 0.8502\,\Sigma A_x$, and if $R_0 = 1.05214r$, then

$$F_p = \frac{0.8502\,\Sigma A_x}{(1.05214r)^2}$$

$$= 0.7683F_c$$

where F_p is the value of the force at the perimeter of the disk when reduced by proximity effect, and F_c is the value of the force at the perimeter of the disk as anticipated by classical theory.

The error factor for a non-computer-simulated spherical model of four elements per radius (that is, a three-dimensional model of 408 elements) is comparatively large, viz., 1.017299, and therefore these results are regarded with some suspicion. Moreover, with only four elements per radius the results may be high up on any trend curve after the fashion of Figure 4 and therefore not in any way representative.

Nevertheless, with the model sphere analyzed $\Sigma A_x \cos \theta_x = 0.8042\,\Sigma A_x$, and $R_0 = 1.186$, from which

$$F_s = \frac{0.8042\,\Sigma A_x}{(1.168r)^2}$$

$$= 0.5895F_o$$

where F_s is the value of the force at the surface of the sphere when reduced by proximity effect, and F_o is the value of the force at the surface of the sphere as anticipated by classical theory. The requirements of an associated study would suggest a feasible value of (F_s/F_o) to be $0.6633 > (F_s/F_o) > 0.5777$, with a possible optimum value of $(F_s/F_o) = 0.6622$.

Although subject to some variation, these results confirm that, following the increased "bluntness" presented by the surface of a sphere, proximity effect is noticeably greater than in the case of the two-dimensional disk.

5. *Conclusions*

A *basic* equation, equation (2.2), has been derived for the solution of inverse square law forces involving multisources or multielement sources in rectilinear systems. It is essentially a modified form of equation (2.1), since equation (2.1) itself involves *unit radius* and is therefore applicable only to orbital systems. In conjunction with this modification it is also necessary to apply the three rules given in 2.2.1, 2.2.2, and 2.2.3 in order to effect the transition from an orbital to a rectilinear system.

This basic equation has been used in a computer evaluation of simple models, spheres and disks, and the following results have been obtained:

5.1.1. The resultant distance between any external reference point and the resultant source of the force exerted by the sphere or disk is that distance between the reference point and the centre of the sphere or disk.

5.1.2. The resultant distance between any internal reference point and the resultant source of the force exerted by the sphere or disk is equal in all cases to the radius of the sphere or disk.

5.1.3. The resultant magnitude of the source external to the surface of the sphere or the perimeter of the disk is equal to the sum of the magnitudes of individual point sources.

5.1.4. The resultant magnitude of the source within the surface of the sphere or within the perimeter of the disk varies directly with the distance from the centre of the sphere or disk, being zero at the centre and maximum at the surface or perimeter.

5.1.5. The resultant force external to the surface of the sphere or to the perimeter of the disk varies with the inverse square of the distance from the centre of the sphere or disk.

5.1.6. The resultant force within the surface of the sphere or within the perimeter of the disk varies directly with distance from the centre of the sphere or disk, being zero at the centre and maximum at the surface or perimeter.

Statements 5.1.1, 5.1.3, 5.1.5, and 5.1.6 are in good agreement with accepted inverse square law force theory, whereas statements 5.1.2 and 5.1.4 are not anticipated by traditional theory as far as the author is aware.

Some discrepancies do exist, and these have been identified as errors introduced by the proximity effects of the source where this is of appreciable size.

Proximity effect, which is not anticipated by traditional theory, effectively reduces the resultant magnitude of the source by a not inconsiderable amount. In general, in the case of a two-dimensional disk,

5.2.1. When $d < r$, error $\simeq -0.15d^3$.
5.2.2. When $d = r$, error $\simeq -0.15$.
5.2.3. When $d > r$, error $\simeq -0.15/d^2$.

It is further shown that this reduction in the resultant magnitude of the source is reflected as a positive error in the resultant distance to the source, this error amounting to some 5% in the extreme case. The net result is that proximity effect reduces the force at the perimeter of the disk to some 77% of its hypothetical value.

Summarizing, the results obtained using the basic equation are consistent with those anticipated by classical theory, and show that the equation offers a realistic solution to the multisource inverse square law force problem in rectilinear systems.

Appendix 1

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PROG1

100 FILES WFILE
110 SCRATCH #1
120 PRINT"HOW MANY DIVISIONS IN RADIUS"
130 INPUT P
140 LET Q=1/P
150 LET Y=-Q
160 LET Y=Y+Q
170 LET X=-Q
180 LET X=X+Q
190 IF X>=1 THEN 160
200 IF (X↑2+Y↑2)>1 THEN 160
210 IF SQR((X+Q)↑2+(Y+Q)↑2)>1 THEN 270
220 LET W=1
230 LET F=X+.5*Q
240 LET G=Y+.5*Q
250 WRITE #1,W;F;G
260 GO TO 180
270 IF Y>(X+.5*Q) THEN 310
280 GOSUB 470
290 IF SQR(X↑2+Y↑2)<1 THEN 180
300 GO TO 160
310 GOSUB 470
320 IF SQR(X↑2+Y↑2)<1 THEN 360
```

```
33Ø LET Y=Y+Q
34Ø IF Y>(1-.5*Q) THEN 83Ø
35Ø LET X = -Q
36Ø LET X=X+Q
365 IF X>=1 THEN 33Ø
37Ø IF (X↑2+Y↑2)>1 THEN 33Ø
38Ø IF SQR((X+Q)↑2+(Y+Q)↑2)>1 THEN 44Ø
39Ø LET W=1
4ØØ LET F=X+.5*Q
41Ø LET G=Y+.5*Q
42Ø WRITE #1,W;F;G
43Ø GO TO 36Ø
44Ø GO SUB 47Ø
45Ø IF SQR(X↑2+Y↑2)<1 THEN 36Ø
46Ø GO TO 33Ø
47Ø LET H=SQR(1-Y↑2)
48Ø LET K=SQR(1-X↑2)
49Ø IF H>(X+Q) THEN 68Ø
5ØØ IF H<=(X+Q) THEN 51Ø
51Ø IF K<=(Y+Q) THEN 61Ø
52Ø IF K>(Y+Q) THEN 54Ø
53Ø REM NUMBER ONE
54Ø LET W=P↑2*Q*(SQR(1-(Y+Q/2)↑2)-X)
55Ø IF SGN(W) =-1 THEN 59Ø
56Ø LET F=X+(SQR(1-(Y+Q/2)↑2)-X)/2
57Ø LET G=Y+.5*Q
58Ø WRITE #1,W;F;G
59Ø RETURN
6ØØ REM NUMBER TWO
61Ø LET W=P↑2*(SQR(1-X↑2)-Y)*(SQR(1-Y↑2)-X)/2
62Ø IF SGN(W)=-1 THEN 66Ø
63Ø LET F=X+Q*W/2
64Ø LET G=Y+Q*W/2
65Ø WRITE #1,W;F;G
66Ø RETURN
67Ø REM NUMBER THREE
68Ø IF K<=(Y+Q) THEN 77Ø
69Ø IF K>(Y+Q) THEN 7ØØ
7ØØ LET W=1-P↑2*((Y+Q)-SQR(1-(X+Q)↑2))*((X+Q)-SQR(1-(Y+Q)↑2))/2
71Ø IF SGN(W)=-1 THEN 75Ø
72Ø LET F=X+Q*W/2
73Ø LET G=Y+Q*W/2
74Ø WRITE #1,W;F;G
```

```
750 RETURN
760 REM NUMBER FOUR
770 LET W=P↑2*Q*(SQR(1-(X+Q/2)↑2)-Y)
780 IF SGN(W)=-1 THEN 820
790 LET F=X+.5*Q
800 LET G=Y+(SQR(1-(X+Q/2)↑2)-Y)/2
810 WRITE #1,W;F;G
820 RETURN
830 PRINT"WFILE COMPLETE"
840 RESTORE #1
850 LET V=0
860 LET T=0
870 IF END #1 THEN 920
880 READ #1,W,X,Y
890 LET V=V+1
900 LET T=T+W
910 GO TO 870
920 LET U=4*T/P↑2
930 PRINT"NUMBER OF ELEMENTS=";V
940 PRINT"AREA OF CIRCLE =";U
950 LET O=U/3.14159
960 PRINT"ERROR=";O
970 END
```

Appendix 2

```
PROG2
100 FILES WFILE
110 PRINT"TYPE IN NUMBER OF ELEMENTS"
120 INPUT T
130 PRINT"TYPE IN DISTANCE FROM CENTRE"
140 INPUT P
150 LET S=0
160 LET U=1
170 GOSUB 500
180 GOSUB 600
190 GOSUB 600
200 GOSUB 500
210 PRINT"SUMMATION OF MASS =",S
220 GOSUB 700
230 GOSUB 800
240 GOSUB 800
250 GOSUB 700
```

```
260 PRINT"RADIUS =",U
270 LET F=S/(U↑2)
280 PRINT"FORCE =",F
290 PRINT"RADIUS/DISTANCE =",U/P
300 STOP
500 RESTORE #1
510 FOR I=1 TO T
520 READ #1,W,X,Y
530 LET R=SQR((X+P)↑2+Y↑2)
540 LET V=(X+P)/R
550 LET M=W*V
560 LET S=S+M
570 NEXT I
580 RETURN
600 RESTORE #1
610 FOR I=1 TO T
620 READ #1,W,X,Y
630 LET R=SQR((−X+P)↑2+Y↑2)
640 LET V=(−X+P)/R
650 LET M=W*V
660 LET S=S+M
670 NEXT I
680 RETURN
700 RESTORE #1
710 FOR I=1 TO T
720 READ #1,W,X,Y
730 LET R=SQR((X+P)↑2+Y↑2)
740 LET V=(X+P)/R
750 LET M=W*V
760 LET L=R↑(M/S)
770 LET U=U*L
780 NEXT I
790 RETURN
800 RESTORE #1
810 FOR I=1 TO T
820 READ #1,W,X,Y
830 LET R=SQR((−X+P)↑2+Y↑2)
840 LET V=(−X+P)/R
850 LET M=W*V
860 LET L=R↑(M/S)
870 LET U=U*L
880 NEXT I
890 RETURN
999 END
```